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**Efficiency Series Paper 1/2011**

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**William Greene, Luis Orea, Alan Wall**



**Departamento de Economía**



**Universidad de Oviedo**

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**A one-stage random effect counterpart of the fixed-effect vector decomposition model with an application to UK electricity distribution utilities\***

**William Greene**

*Dept. of Economics, Stern School of Business, New York University*

**Luis Orea**

*Oviedo Efficiency Group*

*Department of Economics, University of Oviedo*

**Alan Wall**

*Oviedo Efficiency Group*

*Department of Economics, University of Oviedo*

*February 15, 2011*

**Abstract**

To deal with the presence of slowly changing variables, Plümer and Troeger (2007) proposed a fixed effects vector decomposition (FEVD) estimator, which is a three-stage procedure based on the fixed effects estimator. We show that this estimator moves between fixed effect and ordinary least squares depending on how its second stage is specified. This provides an alternative interpretation of the FEVD estimator that allows us to introduce a one-stage counterpart of the FEVD model, which can be viewed as a partial Mundlak (1978) transformation of the random effects model. We illustrate our approach with an application to UK electricity distribution utilities using the same data as Yu et al. (2009). Our application suggests that our estimator and the FEVD yield similar results when the pooled OLS estimates for a given panel are similar to the random effects estimates.

**Keywords:** slowly changing variables, fixed effects vector decomposition, one-stage estimator.

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\* We thank Professor Michael Pollitt (EPRG, University of Cambridge, UK) for providing us the data set on UK electricity distribution utilities, and participants at Workshop on Efficiency and Productivity in Honour of Professor Knox Lovell that took place in Elche (Spain) for their helpful comments.

## 1. Introduction

Fixed effects estimation of production, cost, or demand functions has a long tradition as this model allows to control for unobserved heterogeneity that might be correlated with the explanatory variables. However, estimation of the fixed effect (FE) model may yield implausible parameter estimates when the data contain slowly changing variables, i.e. variables with relatively low within variance. These slowly changing variables are found in many empirical applications. For instance, macro data sets contain aggregates which often move slowly over time and firm-level data often contain slowly changing variables such as labour inputs in farm data, capital stock in transport firms, or energy delivered for electricity distribution companies. To account for the presence of such variables, Plümper and Troeger (2007) proposed what they label the fixed effects vector decomposition (FEVD) estimator. The FEVD estimator is a three-stage approach based on the fixed effect estimator that, in certain circumstances, may provide more precise estimates than the FE estimator in a root mean squared error sense. In the first stage, FE estimates of the panel data model are obtained. In the second stage the estimated unit effects are regressed on the time-invariant variables and the unit means of time-varying variables of the slowly-changing variables, thereby “decomposing” the unit effects into their “observable” and “unobservable” components, the latter corresponding to the residuals from this second stage regression. In the third stage, the model is estimated by pooled OLS using the second-stage residuals as an additional regressor.

Formal analyses of the FEVD estimator have been provided by Greene (2010) and Breusch *et al.* (2010), who show that the FEVD estimator simply reproduces the fixed effects estimates when only strictly time-invariant variables are included in the second stage, i.e., when the unit means of the slowly-changing variables are not included. In this paper we focus on the case where the slowly-changing variables are included in the second stage. The first contribution of our work is that we show that the FEVD estimator moves between FE and OLS depending on how its second stage is specified. In particular, it is shown that the FEVD estimator reproduces the OLS estimates of the parameters of the time-varying and time-invariant variables when the whole set of explanatory variables (i.e. the time-invariant variables and the unit means of the time-varying variables) are included in the second stage. This provides an alternative

interpretation of the FEVD estimator: as we incorporate slowly-changing variables, we move away from FE towards OLS.

Using this interpretation of the FEVD estimator as a shrinkage-type estimator located between FE and OLS, our second contribution is to take advantage of the insights of the FEVD approach to introduce a one-stage counterpart based on the random effects (RE) transformation introduced by Mundlak (1978). This estimator can be viewed as a *partial* random effects Mundlak transformation (REMT), and moves between a consistent but inefficient estimator (FE) and an efficient but inconsistent estimator (RE). Unlike the FEVD, therefore, our estimator moves between two panel data estimators.

To illustrate our approach we provide an application to UK electricity distribution utilities using the same data as Yu *et al.* (2009). Our application suggests that our estimator and the FEVD yield similar results when the pooled OLS estimates for a given panel are similar to the random effects estimates.

## 2. The FEVD estimator

We summarize in this section the main characteristics of the Plümper and Troeger (2007) FEVD estimator. Let us assume we wish to estimate the following model:

$$y_{it} = X_{it}\beta + Z_i\gamma + \alpha_i + \varepsilon_{it} \quad (1)$$

where  $X_{it}$  is a  $K \times 1$  vector of time-varying explanatory variables,  $Z_i$  is a  $P \times 1$  vector of time-invariant explanatory variables,  $\varepsilon_{it}$  is the idiosyncratic error term, and  $\alpha_i$  captures the effect of unobserved time-invariant individual characteristics.

The three stages of the FEVD procedure can be neatly presented using the notation in Breusch *et al.* (2010), which will also facilitate the discussion in the next section of the relationship of the FEVD estimator with both OLS and FE. In particular, let  $D = I_N \otimes \iota_T$  be the matrix of dummy variables indicating group membership, where  $I_N$  is an  $N \times N$  identity matrix and  $\iota_T$  is a  $T \times 1$  vector of ones. We use the projection matrix for

$D$ , i.e.  $P_D = D(D'D)^{-1}D'$ , to get a vector of group means, and  $Q_D = I_{NT} - P_D$  to reproduce the within-group variation.

In the first stage, a FE regression is performed using the within transformation, so that the individual effects  $\alpha_i$  and the time-invariant variables  $Z_i$  are removed:

$$Q_D y = Q_D X \beta + Q_D \varepsilon \quad (2)$$

where  $Q_D y = \{y_{it} - \bar{y}_i\}$ ,  $Q_D X = \{X_{it} - \bar{X}_i\}$ , and  $Q_D \varepsilon = \{\varepsilon_{it} - \bar{\varepsilon}_i\}$ . The moment condition corresponding to this FE regression is:

$$(y - X\beta)' Q_D X = 0 \quad (3)$$

and the estimated unit effects  $\hat{\alpha}_i$  are  $\hat{\alpha}_i = P_D(y - X b_{FE})$  where  $b_{FE}$  is the fixed effects estimate of (2).

In the second stage, these estimated unit effects are regressed on the observed time-invariant variables and the group means of a subset  $X_J$  of the  $K$  time-varying variables, where the  $J < K$  variables in  $X_J$  are those with relative low within-variance, i.e. slowly changing variables. As  $P_D(y - X b_{FE})$  is regressed on  $P_D X_J b_2$  and  $Zg$ , the residuals from this regression are  $P_D(y - X b_{FE}) - P_D X_J b_2 - Zg$  and the corresponding moment conditions of this second stage can be written as:

$$(P_D(y - X b_{FE}) - P_D X_J b_2 - Zg)' P_D X_J = 0 \quad (4a)$$

$$(P_D(y - X b_{FE}) - P_D X_J b_2 - Zg)' Z = 0 \quad (4b)$$

where  $b_2$  and  $g$  are parameters to be estimated. Thus, the unit effects are decomposed into a part explained by the available between-unit information contained in  $Z$  and the subset  $J$  of rarely changing variables, and an unexplained part which corresponds to the residual from this second stage regression. The group-average residuals,  $h$ , from this regression are:

$$h = P_D(y - X b_{FE} - X_J b_2 - Zg) \quad (5)$$

In the third and final stage, the full model is run using pooled OLS without the unit effects but including the group-average residuals  $h$  from the second stage, yielding the final FEVD estimates. The moment conditions are:

$$(y - X\beta - Z\gamma - h\delta)'[X, Z, h] = 0 \quad (6)$$

Using the moment-condition representation above in the next section we show that this FEVD estimator can be interpreted as a shrinkage-type estimator which is located between FE (a consistent but higher variance estimator) and OLS (an inconsistent but lower variance estimator). The crucial issue here is the variables that are included in the second step of the FEVD estimator.

### 3. Relationship between the FEVD estimator and both OLS and FE

When only the strictly time-invariant variables,  $Z$ , are included in the second stage, Greene (2010) and Breusch *et al.* (2010) prove the following: (i) the estimated coefficients of the time-varying variables in the third stage of the FEVD are exactly the same as in FE ( $\beta = \beta_{FE}$ ); (ii) the estimates of the coefficients of the time-invariant variables are the same as those from the second stage ( $\gamma = g$ ); and (iii)  $\delta = 1$ . As  $X_j$  in the second stage above contains no elements ( $J = 0$ ), the group average residuals from this stage are simply :

$$h = P_D(y - Xb_{FE} - Zg) \quad (7)$$

To prove that the FEVD estimator simply reproduces the fixed effects estimates when only time-invariant variables are included in the second stage, Breusch *et al.* (2010, Theorem 1) verified that the moment conditions (6) are satisfied at  $\beta = b_{FE}$ ,  $\gamma = g$ , and  $\delta = 1$ . This requires that

$$(y - Xb_{FE} - Zg - h)'[X, Z, h] = 0 \quad (8)$$

Substituting in the definition of  $h$  from (7) and gathering terms, this simplifies to

$$(y - Xb_{FE})'Q_D[X, Z, h] = 0 \quad (9)$$

It is straightforward to see that the first set of conditions in (9) must be satisfied, since it is identical to the moment condition (3) that defines  $b_{FE}$ . The other two set of conditions must be also satisfied since both  $Z$  and  $h$  are time-invariant.

We next analyze the opposite situation, assuming that the group means of all the time-varying variables are included as regressors in the second stage ( $J = K$ ). Using the above moment-condition representation, we demonstrate that the FEVD estimator collapses to OLS, which can be represented by the following moment conditions:

$$(y - Xb_{OLS})'X = 0 \quad (10)$$

where we have dropped  $Z$  for notational ease and the OLS estimate of (10) is denoted by  $b_{OLS}$ .<sup>1</sup>

To prove that the FEVD estimator reproduces the OLS estimates in this case, we follow the same methodological strategy as above, i.e., we will verify that the moment conditions (6) are satisfied at  $\beta = b_{OLS}$ , and  $\delta = 1$  when the group means of all the time-varying variables are included as regressors in the second stage. This is equivalent to assuming that  $J = K$  and hence the moment conditions in the second stage can be written as:

$$(P_D(y - Xb_{FE}) - P_D X b_2)'P_D X = [P_D(y - Xb_{FE} - Xb_2)]'P_D X = 0 \quad (11)$$

The group-average residuals  $h$  from this regression are:

$$h = P_D(y - Xb_{FE} - Xb_2) \quad (12)$$

Taken into account that  $P_D'P_D = P_D$ , the moment conditions in (11) can be written as:

$$(y - X(b_{FE} + b_2))'P_D X = 0 \quad (13)$$

Gathering the sets of both first and second-stage coefficients and defining  $\theta = b_{FE} + b_2$ , we finally get that:

$$(y - X\theta)'P_D X = 0 \quad (14)$$

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<sup>1</sup> The same results are obtained by including  $Z$  in (10). It is worth noting that except when we work with the FE estimator we can interpret  $X$  as a vector of *both* time-varying and time-invariant variables without any change in the results.

Note that this set of moment conditions coincides with those corresponding to the Between estimator, which estimates (1) by OLS using the individual group means, i.e.  $P_D y$  and  $P_D X$ . The moment conditions of the Between estimator are:

$$(P_D y - P_D X b_{BET})' P_D X = 0 \quad (15)$$

where  $b_{BET}$  stands for the Between coefficients. These moment conditions are equivalent to:

$$[P_D (y - X b_{BET})]' P_D X = (y - X b_{BET})' P_D X = 0 \quad (16)$$

It can be seen that the moment conditions in (14) and (16) are the same, from which it follows that  $\theta = b_{FE} + b_2 = b_{BET}$ . That is, the parameter estimates in the second stage,  $b_2$ , correspond to the difference between the Between and Within estimates, i.e.  $b_2 = b_{BET} - b_{FE}$ .<sup>2</sup>

The previous result implies that the group-average residuals (12) can be written as:

$$h = P_D (y - X b_{FE}) - P_D X (b_{BET} - b_{FE}) = P_D (y - X b_{BET}) \quad (17)$$

Therefore, the group-average residuals in the second stage do not actually depend on coefficients estimated in the first stage of the FEVD procedure using the FE estimator, but on between coefficients that have not been explicitly estimated. Note, in addition, that if we take into account that

$$b_{BET} = [(P_D X)' (P_D X)]^{-1} (P_D X)' (P_D y) = [X' P_D X]^{-1} X' P_D y \quad (18)$$

the group-average residuals in (17) can be written as:

$$h = P_D y - P_D X [X' P_D X]^{-1} X' P_D y \quad (19)$$

If we next pre-multiply by  $X'$ , we get:

$$X' h = X' P_D y - X' P_D X [X' P_D X]^{-1} X' P_D y = X' P_D y - X' P_D y = 0 \quad (20)$$

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<sup>2</sup> The same results are obtained if we explicitly work with time-invariant variables in equation (15). Indeed, when time-invariant variables are included, the corresponding moment conditions of the second stage can be written as  $(y - X\theta - Zg)' P_D X = 0$  and  $(y - X\theta - Zg)' Z = 0$ . On the other hand, the moment conditions of the Between estimator, which now implies regressing  $P_D y$  against  $P_D X$  and  $Z$ , can be written as  $(y - X b_{BET} - Z g_2)' P_D X = 0$  and  $(y - X b_{BET} - Z g_2)' Z = 0$ , where  $g_2$  stands for the between coefficients of the time-invariant variables. Again both sets of moment conditions are the same, and  $\theta = b_{FE} + b_2 = b_{BET}$ , and  $g = g_2$ .



Armed with the above results, we can demonstrate that the moment conditions in the third stage are satisfied at  $\beta = b_{OLS}$ , and  $\delta = 1$ . Ignoring the time-invariant variables, these moment conditions simplify to

$$(y - X\beta - h\delta)'[X, h] = 0 \quad (21)$$

If the set of moment conditions in (21) are satisfied when  $\beta = b_{OLS}$ , and  $\delta = 1$ , this implies that

$$(y - Xb_{OLS} - h)'X = 0 \quad (22)$$

$$(y - Xb_{OLS} - h)'h = 0 \quad (23)$$

Taking the transpose in (22) we get

$$X'(y - Xb_{OLS} - h) = X'(y - Xb_{OLS}) - X'h = 0 \quad (24)$$

Since  $X'h = 0$ , the above set of conditions must be satisfied, since it is identical to the moment condition (10) that defines  $b_{OLS}$ .

Taking the transpose in (23) we get

$$h'(y - Xb_{OLS} - h) = h'(y - h) - h'Xb_{OLS} = 0 \quad (25)$$

Using the fact that  $X'h = 0$ , and substituting for  $h$  from (17), the above condition is identical to

$$h'(y - h) = (y - Xb_{BET})'P_D'(y - P_D y + P_D Xb_{BET}) = 0 \quad (26)$$

As  $P_D' = P_D$ , this simplifies to

$$h'(y - h) = (y - Xb_{BET})'P_D Xb_{BET} = 0 \quad (27)$$

The above condition must be satisfied since it is identical to the moment condition (15) that defines  $b_{BET}$ , showing that the FEVD estimator reproduces the OLS estimates when the whole set of time-varying variables are included as regressors in the second stage.

In summary, this section has shown that the FEVD estimator can be viewed as an estimator located between FE and OLS. This is illustrated in Figure 1. When only

strictly time-invariant variables are included in the second stage of the FEVD estimator, it reproduces the FE estimates. As we incorporate the group means of the time-varying variables, we move away from FE towards OLS, arriving at OLS estimates when the group means of all the time-varying variables are included.

[INSERT FIGURE 1 HERE]

The crucial issue with the FEVD estimator is therefore what variables to include in the second stage. Strictly time-invariant characteristics will obviously be included and variables with sufficiently low within-variance should also be included. Plümper and Troeger (2007) carry out Monte Carlo simulations to provide the conditions under which a time-varying variable should be included in the second stage. Using the root mean squared error as their criterion, they find that the decision to treat a slowly changing variable as time-varying or time-invariant depends on the correlation between the variable and the unobserved heterogeneity and the ratio of the between to within variance. For a correlation of 0.3 between the variable and the unit heterogeneity, a between-to-within ratio of approximately 1.7 is sufficient for the FEVD estimator to outperform FE. When the correlation rises to 0.5, the between-to-within ratio rises to about 2.8. While this correlation is unobservable, the inclusion of additional variables in  $z$  will reduce the potential for correlation, and Plümper and Troeger (2007) suggest that a between-to-within ratio of 2.8 is sufficient to justify the inclusion of the variable in the second stage.

The importance of the between-to-within variance as a criterion for the inclusion of time-varying variables in the second stage is thus clear. The aim is to maximize the use of between variation for those variables with relatively low within variation, and we include the group means of more time-varying variables we move away from FE to OLS. Alternatively, we move away from a consistent but higher variance estimator (FE) towards an inconsistent but lower variance estimator (OLS), and it is in this sense that the FEVD can be interpreted as a shrinkage estimator where the criterion for leaning more towards one or the other depends on the between-to-within variance of the time-varying variables and their subsequent inclusion in the second stage.

#### 4. A partial Random Effects Mundlak transformation

We take advantage of the preceding interpretation of the FEVD estimator to introduce a one-stage counterpart of the estimator that can also be interpreted as a shrinkage estimator, this time between FE and RE. Our model is based on the random effects Mundlak transformation (REMT) introduced by Mundlak (1978).<sup>3</sup> Mundlak showed that the FE parameter estimates can be obtained from the RE model by simply adding the individual means of *all* time-varying variables as explanatory variables. In the REMT, the individual effects  $\alpha_i$  in (1) are substituted by

$$\alpha_i = \omega_i + \sum_{k=1}^K \rho_k \bar{x}_{ki}, \quad \omega_i \sim N(0, \sigma_\omega^2) \quad (28)$$

Replacing the unit effects in (1) with (28), the REMT model can be expressed as:

$$y_{it} = \omega_i + \sum_{k=1}^K \rho_k \bar{x}_{ki} + \sum_{k=1}^K \beta_k x_{kit} + \sum_{m=1}^M \gamma_m z_{mi} + \varepsilon_{it} \quad (29)$$

where it is assumed that  $z_i$  and  $\alpha_i$  are uncorrelated.

REMT corrects for endogeneity at the cost of discarding all between information, which is of course why it yields FE estimates as it only uses the within information of the time-varying variables.<sup>4</sup> Our proposal is to introduce *some* but not all of the unit means as additional regressors in the RE model. This model can therefore be labelled as a Partial REMT. This will permit us to take advantage of the between information of the variables whose unit means are excluded as regressors, which will lead to lower-variance estimates at the cost of not controlling for the possible endogeneity of these variables. As in the FEVD, therefore, the issue is one of trading bias for efficiency.

As shown in Figure 2, if we introduce the unit means of only a subset  $J$  of the  $K$  time-varying variables, where the variables in  $J$  are those with sufficiently high within-variance for reasons that will become clear below, we again arrive at an estimator which is located between a consistent but higher variance estimator (FE) and an inconsistent but lower variance estimator (RE).

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<sup>3</sup> See also Chamberlain (1980).

<sup>4</sup> This can be seen by noting that the REMT model (29) can be rewritten as:

$$y_{it} = \omega_i + \sum_{k=1}^K (\rho_k + \beta_k) \bar{x}_{ki} + \sum_{k=1}^K \beta_k (x_{kit} - \bar{x}_{ki}) + \sum_{m=1}^M \gamma_m z_{mi} + \varepsilon_{it}.$$

[INSERT FIGURE 2 HERE]

Replacing the unit effects in (1) with a modified version of (28) where only  $J$  unit means are included so that  $\alpha_i = \omega_i + \sum_{k=1}^{J < K} \rho_k \bar{x}_{ki}$ , the partial REMT model can finally be expressed as:

$$y_{it} = \omega_i + \sum_{k=1}^{J < K} \rho_k \bar{x}_{ki} + \sum_{k=1}^K \beta_k x_{kit} + \sum_{m=1}^M \gamma_m z_{mi} + \varepsilon_{it} \quad (30)$$

Which unit means should be included in  $J$ ? Recall that Plümer and Troeger (2007) included unit means in their second stage to make use of between information for variables with relatively low within variation so that the variables included were those with high between-to-within variance ratios. The more variables included in this second stage, the further away from FE and the closer the estimator gets to OLS. In our partial REMT we are going in the opposite direction, towards FE: the more unit means we introduce into (30), the less between information we use and the more we rely on within information. As a criterion for choosing our unit means, ideally we will only discard between information when we are left with sufficient within information to satisfactorily estimate the parameter. Our criterion is thus the inverse of that of the FEVD: the unit means to be introduced will be those of variables with relative low between-to-within variance ratios.

By including the unit means of only those variables with sufficiently high within-variance, the coefficients of slowly changing variables are biased, as in the FEVD model, but have lower variances than estimates from the FE model. Note, however, that these estimates are obtained here in a one-stage regression, while the FEVD model requires estimating three equations.

## 5. Empirical Illustration

We illustrate our approach with an application to UK electricity distribution utilities using the dataset of Yu *et al.* (2009) on 12 distribution networks in the UK for the 1995/96 to 2002/03 period. This dataset is particularly appropriate for our purposes as many crucial determinants of utility costs such as the energy delivered or the number of customers are persistent or slowly changing variables. Moreover, there are many

characteristics of the electricity distribution sector, such as geography, weather conditions, network characteristic, etc. that affect production costs but which are not observed (Farsi and Filippini, 2004) so that individual firm effects need to be modeled. . Table 1 reports the summary statistics of the data used. All monetary variables are expressed in 2003 real terms.

[INSERT TABLE 1 HERE]

To illustrate our approach we estimate a simple cost function than can be written as:

$$TotalCost_{it} = \alpha_i + \beta_1 \cdot ENERGY_{it} + \beta_2 \cdot EPR_{it} + \beta_3 \cdot CML_{it} + \varepsilon_{it} \quad (31)$$

where *TotalCost* includes capital and operational costs and the opportunity cost of network energy losses, following Jamasb *et al.* (2010). The output variable *ENERGY* is the energy delivered;<sup>5</sup> *EPR* is the price for network energy losses; and *CML* is a measure of service quality, measured by the customers minutes lost. The between and within standard deviations are shown in Table 2.

[INSERT TABLE 2 HERE]

The estimated coefficients and the heteroskedasticity-robust standard errors from the OLS, FE and FEVD estimators are shown in Table 3.

[INSERT TABLE 3 HERE]

All coefficients in the OLS model have their expected signs. Thus, the coefficients of energy delivered and input price are positive and statistically significant and the coefficient of customer minutes lost is negative, suggesting a positive marginal cost of quality improvements. However, Jamasb *et al.* (2010) note that the quality of service variable may be correlated with average weather conditions, and hence the estimated marginal cost of quality improvements is likely to be downward biased. They examine how to address this issue when weather data is available. Here we will try to control for this endogeneity problem using “adjusted” FE and RE estimators.

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<sup>5</sup> Customer numbers and units of energy delivered are the most commonly-used outputs in the benchmarking of distribution network utilities. Given that the statistical correlation between these two outputs is large (over 97%), we only present our parameter estimates using energy delivered as a unique output. The specification of the cost function that uses the energy delivered as output appears more appropriate as our dependent variable includes the cost of energy losses.

The low precision of the FE estimator in the present application is clearly illustrated by the fact that the coefficient of the energy delivered variable is negative and not statistically significant. The reason is that the within variation of this variable is much lower than the between variation (see Table 2). We therefore estimate a FEVD model using the variable *ENERGY* as a regressor in the second stage. From Table 3 it can be seen that all coefficients again have their expected signs. The coefficient of the residual from the second stage ( $\theta_i$ ) is close to unity, which is expected.<sup>6</sup> Note also that the parameter estimate for CML increases in absolute terms to 0.235. This value is quite similar to the marginal cost of quality improvement obtained in Jamasb *et al.* (2010) where weather variables are included to control for the endogeneity of the quality services variable.

The estimates from the Between and FE estimators and the second stage of the FEVD estimator when the group means of all the time-varying variables are included are shown in Table 4. This illustrates the result from Section 3 that the estimates on the group means of the time-varying from the second stage of the FEVD are the difference between the estimates from the Between and FE estimators.

[INSERT TABLE 4 HERE]

Table 5 shows the estimated coefficients using RE-based models. All coefficients in the RE model have the expected signs. Interestingly, the OLS and RE models yield the same estimates as the variance of the unit effects was found to be equal to zero in the RE model. This was despite the fact that in the FE model we find evidence of unit effects and a Hausman test rejected RE (see Table 3). Hence, both the FEVD and partial REMT estimators are effectively located between the same estimators insofar as OLS and RE are the same. This provides an ideal situation to compare both estimators. The second model in Table 5 is the REMT which reproduces the FE parameter estimates as it includes the individual means of *all* cost determinants (i.e.  $\overline{ENERGY}$ ,  $\overline{EPR}$  and  $\overline{CML}$ ) as explanatory variables.

[INSERT TABLE 5 HERE]

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<sup>6</sup> Recall that Breusch *et al.* (2010) and Greene (2011) have shown that when only time-invariant variables are included in the second stage the coefficient is exactly unity.

For the partial REMT model we include the individual means of those variables with the lowest between-to-within variance ratios, i.e.  $\overline{EPR}$  and  $\overline{CML}$ .<sup>7</sup> The point estimate of the only slowly changing variable in our application (*ENERGY*) is again reasonable from an economic perspective. The partial REMT model produces quite similar estimates to the FEVD, as is to be expected given the similarity of the RE and OLS estimates.

#### 4. Conclusions

In this paper we have shown that the fixed-effect vector decomposition (FEVD) estimator introduced by Plümper and Troeger (2007) can be interpreted as an estimator lying between a consistent but higher variance estimator (FE) and a biased but lower variance estimator (OLS). The FEVD estimator moves from FE towards OLS as we incorporate the group means of time-varying variables in the second stage of the FEVD estimator. The criterion for including time-varying variables in the second stage is related to the between-to-within variance of the time-varying variables. Using the insights of the FEVD estimator, we propose a one-stage counterpart which can be viewed as a partial random effects Mundalk (REMT) transformation and we have illustrated the estimator with an application to UK electricity distribution utilities. Our results show that the partial REMT permits more reasonable coefficients of variables with low within variance than FE and for the data set used the estimates are quite close to those of the FEVD. We believe that our model offers an interesting alternative to FEVD in the presence of slowly changing variables and variables with low between-to-within variance ratios. As our model lies between FE and RE, the better the RE estimates the better the partial REMT. When the RE and OLS estimates are close, the partial REMT can be expected to produce similar estimates to FEVD, with the difference that the partial is a mix of two pure panel data estimators whereas the FEVD is a mix of a panel estimator (FE) and a non-panel estimator (OLS).

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<sup>7</sup> Note from Table 2 that the inclusion of  $\overline{EPR}$  will lead to little loss of between information. Including  $\overline{CML}$  will entail a loss of substantial between information but this still leaves a lot of within information to work with as can be seen by the fact that the within variance is higher than the between variance.

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**Table 1:** Descriptive Statistics (96 Observations)

Variable	Unit	Mean	Std. dev.	Min	Max
<i>TotalCost</i>	Million £	243.99	85.66	88.16	449.99
<i>ENERGY</i>	Thousand GWh	20.67	7.26	7.492	36.262
<i>CML</i>	Million Minutes	163.75	76.58	60.67	670.58
<i>EPR</i>	Thousand £	43.79	12.93	25.19	77.06

**Table 2.** Between and within standard deviations

Variable	Between	Within	Between-to-within ratio
<i>ENERGY</i>	7.43	1.24	5.98
<i>EPR</i>	3.38	12.52	0.27
<i>CML</i>	51.34	58.50	0.88

**Table 3.** Cost function parameter estimates: OLS, FE and FEVD

Variable	OLS		FE		FEVD	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
<i>ENERGY</i>	9.770	0.779	-3.189	4.890	10.179	0.760
<i>EPR</i>	4.232	0.304	3.341	0.349	4.120	0.298
<i>CML</i>	-0.155	0.054	-0.178	0.076	-0.235	0.061
<i>Constant</i>	243.99	4.231	243.99	3.917	243.99	3.913
$\theta$	-	-	-	-	1.009	0.263
Hausman test (d.o.f.)			27.55 (3)			
<i>F</i> -test: $\alpha_i = 0$ ( <i>p</i> -value)			2.39 (0.01)			
FEVD 2 <sup>nd</sup> stage ( <i>Dep. Var.</i> = $\alpha_i$ )						
$\overline{ENERGY}$					13.130	0.2415
$R^2$					0.969	

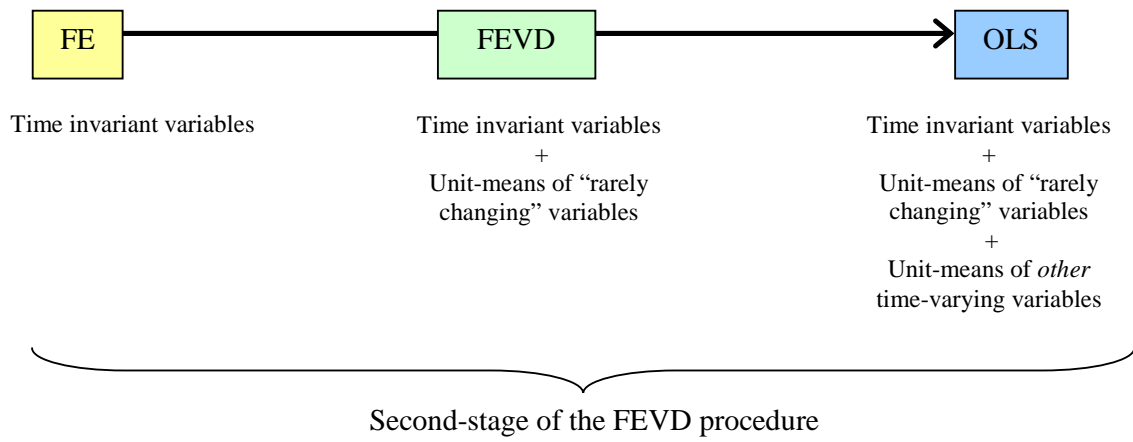
**Table 4.** Between, FE and second stage FEVD estimates

Variable	BETWEEN		FE		FEVD 2 <sup>nd</sup> Stage	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
<i>ENERGY</i>	8.626	0.930	-3.189	4.890	11.815	0.274
<i>EPR</i>	5.569	1.346	3.341	0.349	2.228	0.397
<i>CML</i>	0.151	0.126	-0.178	0.076	0.329	0.037
<i>Constant</i>	243.99	3.885	243.99	3.917	-	-

**Table 5.** Cost function parameter estimates: RE-based models.

Variable	RE		REMT		Partial REMT	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
<i>ENERGY</i>	9.770	0.779	-3.189	5.278	8.129	0.876
<i>EPR</i>	4.232	0.304	3.341	0.378	4.045	0.309
<i>CML</i>	-0.155	0.054	-0.178	0.068	-0.252	0.062
$\overline{ENERGY}$	-	-	11.815	5.277	-	-
$\overline{EPR}$	-	-	2.228	1.071	1.267	1.053
$\overline{CML}$	-	-	0.329	0.163	0.455	0.150
<i>Constant</i>	243.99	4.231	243.99	3.914	243.99	4.036

**Figure 1.** FEVD estimator and its anchoring estimators



**Figure 2.** Partial adjusted RE estimator and its anchoring estimators

